Video 8

Sunday, October 4, 2020 9:56 AM

Binary linear classification

Classification task: predict a discrete-valued target

Binary: only 2 possible targets $t \in \{0, 1\}$

negative examples (0), positive examples (1)

Linear: $z = \vec{w}^T \vec{x} + b$ } identical to linear regression

$$y = \begin{cases} 1 & \text{if } z \geq r \\ 0 & \text{if } z < r \end{cases}$$

y: prediction of linear classifier; r: threshold

Eliminate the threshold r, $\vec{w}^T\vec{x} + b \geq r \iff \vec{w}^T\vec{x} + \underbrace{(b - r)}_{b'} \geq 0$

From now on $r = 0$

Eliminate the bias b,

Add a dummy feature x_0 set to value of 1

Then, the corresponding weight value $w_0 = b$

$$\vec{x} = \begin{pmatrix} x_1 \\ \vdots \\ x_D \end{pmatrix} \quad \vec{w} = \begin{pmatrix} w_1 \\ \vdots \\ w_D \end{pmatrix} \iff \vec{x} = \begin{pmatrix} x_0 = 1 \\ x_1 \\ \vdots \\ x_D \end{pmatrix} \quad \vec{w} = \begin{pmatrix} b = w_0 \\ w_1 \\ \vdots \\ w_D \end{pmatrix}$$

b

Simplified linear classifier: $z = \vec{w}^T\vec{x}$

$$y = \begin{cases} 1 & \text{if } z \geq 0 \\ 0 & \text{if } z < 0 \end{cases}$$

Example 1 NOT $x_1 \in \{0, 1\}$

use linear classifier $t = \neg x$

dummy feature used to eliminate the bias

x_0	x_1	$t = \neg x_1$	
1	0	1	← first training ex $x^{(1)}, t^{(1)}$
1	1	0	← 2^{nd} training ex $x^{(2)}, t^{(2)}$

$x^{(2)} = (x_0^{(2)}, x_1^{(2)})^T$

$w_0 x_0^{(1)} + w_1 x_1^{(1)} \geq 0$ since $t^{(1)} = 1$

$w_0 x_0^{(2)} + w_1 x_1^{(2)} < 0$ since $t^{(2)} = 0$

$$\begin{cases} w_0 \cdot 1 + w_1 \cdot 0 \geq 0 \\ w_0 \cdot 1 + w_1 \cdot 1 < 0 \end{cases} \Rightarrow \begin{cases} w_0 \geq 0 \\ w_0 + w_1 < 0 \end{cases}$$

Select values: $w_0 = 1$, $w_1 = -2$ $\quad w_0 \geq 0$ ✓ $\quad w_0 + w_1 = -1 < 0$ ✓

For pb of NOT: linear classifier with parameters $\vec{w} = \begin{pmatrix} w_0 \\ w_1 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$

<u>problem is linearly classifiable</u>

Example 2 AND x_1, x_2 are the two input features $x_1, x_2 \in \{0, 1\}$

$t = x_1 \wedge x_2$ is the target $t \in \{0, 1\}$

dummy variable (x_0), 2 input features (x_1, x_2)

x_0	x_1	x_2	$t = x_1 \wedge x_2$	
1	0	0	0	← 1^{st} train ex
1	0	1	0	← 2^{nd} ——
1	1	0	0	← 3^{rd} ——
1	1	1	1	← 4^{th} ——

$w_0 \cdot x_0^{(1)} + w_1 \cdot x_1^{(1)} + w_2 x_2^{(1)} < 0$ since $t^{(1)} = 0$

$w_0 \cdot 1 + w_1 \cdot 0 + w_2 \cdot 0 < 0$

$$\begin{cases} w_0 < 0 & \text{(constraint derived from } 1^{st} \text{ train ex)} \\ w_0 + w_2 < 0 & \text{since } t^{(2)} = 0 \quad \text{(constraint derived from } 2^{nd} \text{ train ex)} \\ w_0 + w_1 < 0 & \text{since } t^{(3)} = 0 \quad (\ldots 3^{rd}) \\ w_0 + w_1 + w_2 \geq 0 & \text{since } t^{(4)} = 1 \quad (\ldots 4^{th}) \end{cases}$$

Select values $\begin{cases} w_0 = -1.5 \\ w_1 = 1 \\ w_2 = 1 \end{cases}$

$\Rightarrow$ AND is a linearly classifiable binary task.

Example 3 XOR

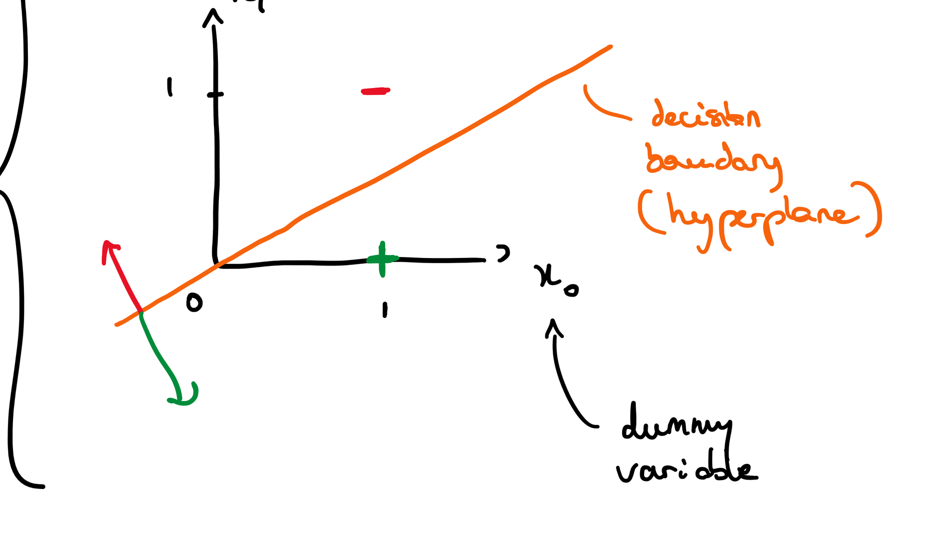
Input vs weight space visualization

$t = 1$ +
$t = 0$ −

Example 1: NOT

x_0	x_1	$t = \neg x_1$
1	0	1
1	1	0

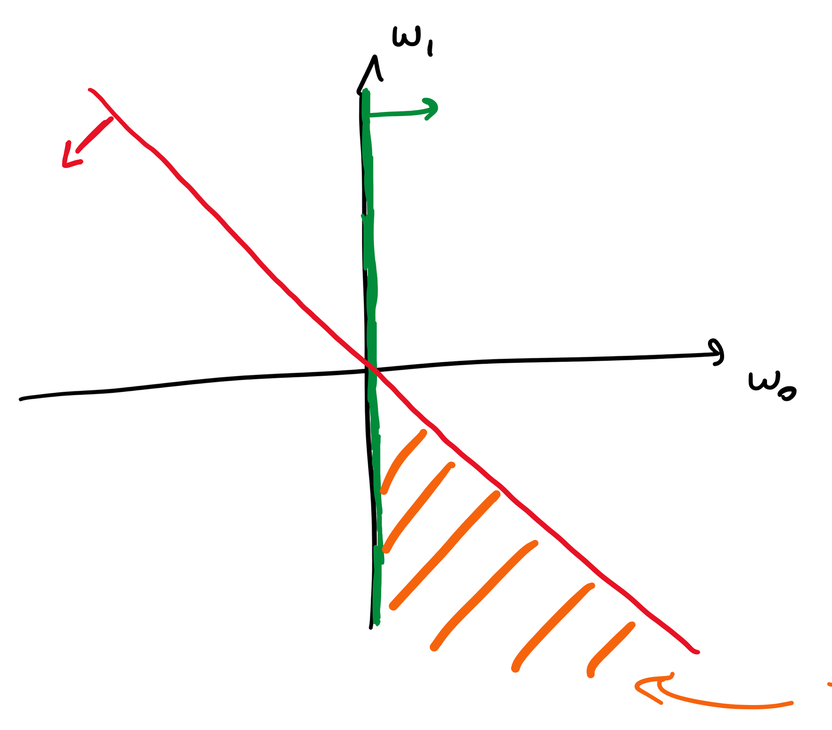
INPUT SPACE



Decision Boundary: $\vec{w}^T x = 0$

$w_0 > 0$

$w_0 + w_1 \leq 0$



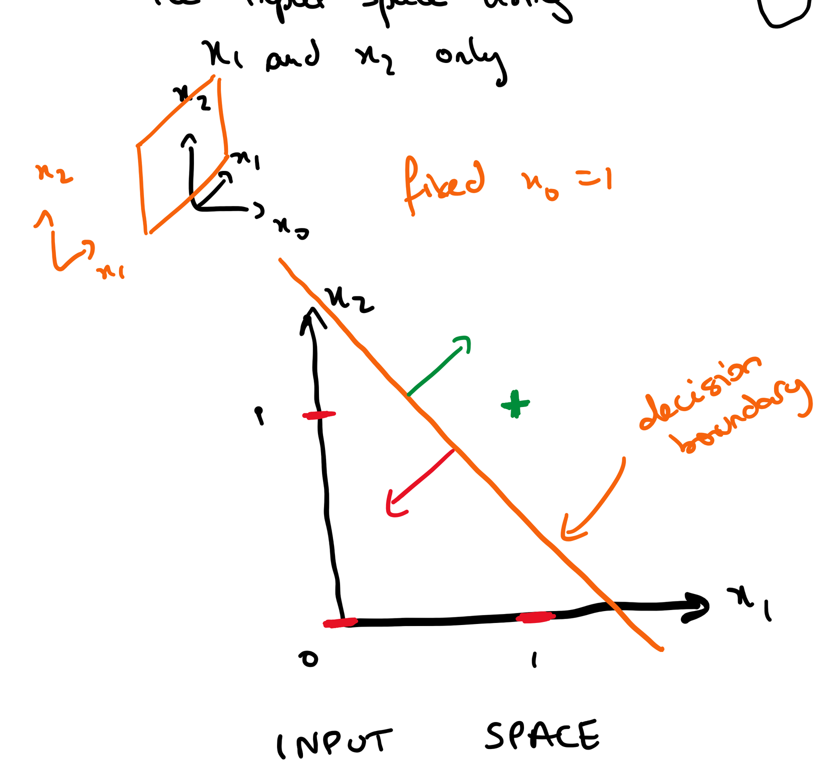
Example 2: AND

x_0	x_1	x_2	$t = x_1 \wedge x_2$
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

$$\begin{cases} w_0 < 0 \\ w_0 + w_2 < 0 \\ w_0 + w_1 < 0 \\ w_0 + w_1 + w_2 \geq 0 \end{cases}$$

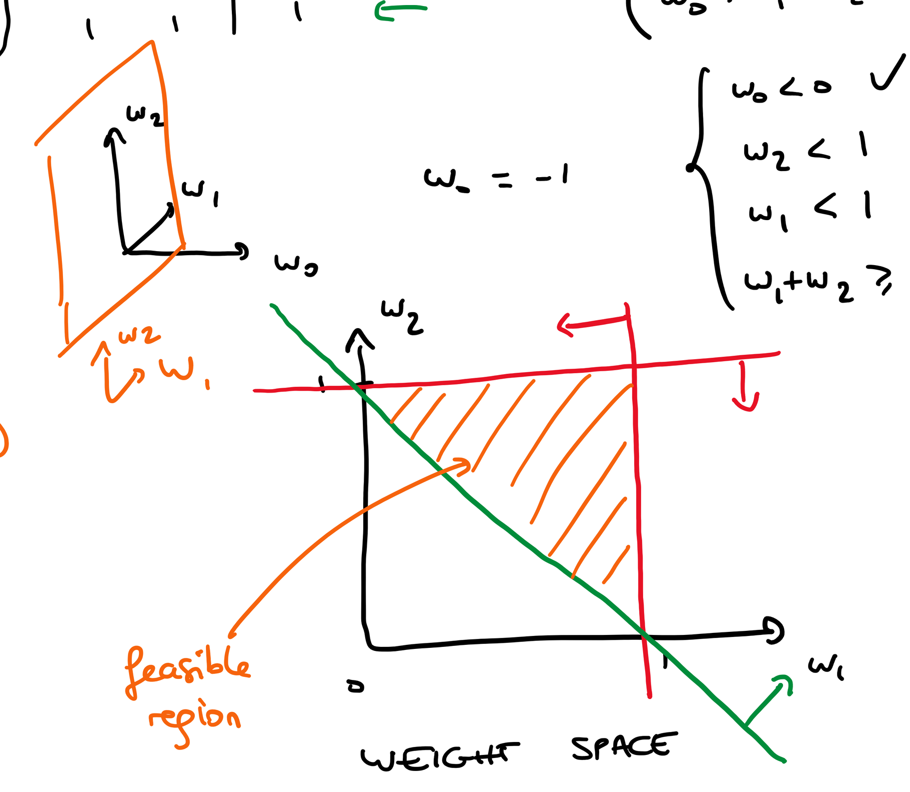
Since $x_0 = 1$ always, we're going to visualize the input space using x_1 and x_2 only

fixed $x_0 = 1$



$w_0 = -1$

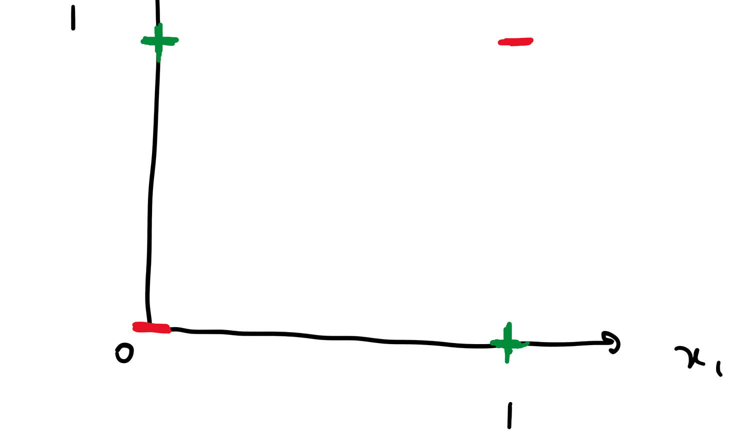
$$\begin{cases} w_0 < 0 \ ✓ \\ w_2 < 1 \\ w_1 < 1 \\ w_1 + w_2 \geq 1 \end{cases}$$



Example 3 XOR

(dummy) x_0	x_1	x_2	$t = x_1 \oplus x_2$
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

INPUT SPACE



$x_0 = 1$